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LETTER TO THE EDITOR

Quantum D = 4 Poincaré superalgebra

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Abstract. The κ-deformation of D = 4 Poincaré algebra is extended to the $N=1$ D = 4 Poincaré superalgebra. By the contraction of real Hopf superalgebra $U_q(OSp(1|4))$ (q real) we obtain real Hopf algebra $U_\kappa(\mathcal{P}_{4;1})$ ($\mathcal{P}_{4;1}$) is written in the form which in the limit of vanishing fermionic generators (supercharges) reduces to the κ-Poincaré algebra $U_\kappa(\mathcal{P}_4)$ ($\mathcal{P}_4$ describes $D=4$ Poincaré algebra) proposed by Lukierski, Nowicki and Ruegg.

Recently the formalism of quantum (super) groups and quantum (super) algebras ([1–4]; for 'super' case see [5–6]) has been applied to describe the deformations of the fundamental D = 4 space–time symmetries [7–17] as well as their supersymmetric extensions [12, 18–20]. The first 'genuine' quantum deformation of D = 4 Poincaré algebra (i.e. taking the form of a real Hopf algebra) was given in [15], where the κ-Poincaré algebra with standard real form was derived§. In this letter we shall 'supersymmetrize' the scheme presented in [15] by replacing the contraction

$$U_q[O(3,2)] \underset{\left[\begin{smallmatrix} q\to 1 \\ R\to\infty \end{smallmatrix}\right]_\kappa}{\longrightarrow} U_\kappa(\mathcal{P}_4) \tag{1a}$$

by the following one

$$U_q[OSp(1|4)] \underset{\left[\begin{smallmatrix} q\to 1 \\ R\to\infty \end{smallmatrix}\right]_\kappa}{\longrightarrow} U_\kappa(\mathcal{P}_{4;1}) \tag{1b}$$

§ In our earlier work [7] we obtained the κ-deformation of Poincaré algebra with non-standard real form.

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where in both cases the limit occurring in (1*b*) is described by the contraction $R \to \infty$ under the assumption that $q = \exp(1/2\kappa R)$, i.e.†

$$\begin{bmatrix} q \to 1 \\ R \to \infty \end{bmatrix}: \qquad \lim_{R\to\infty} R \ln q = \frac{1}{2\kappa}. \tag{2}$$

As a result we obtain the D = 4 N = 1 κ-super Poincaré algebra, with the following basic properties:
(i) The standard D = 4 supersymmetry relation‡ ($\bar{Q} \equiv Q^T \gamma_0$; $a, b = 1, 2, 3, 4$)

$$\{\bar{Q}_a, Q_b\} = D_{ab} = (\gamma^\mu)_{ab} P_\mu \tag{3a}$$

with $D^2 = P_0^2 - \boldsymbol{P}^2 = P_\mu P^\mu$ is replaced by

$$\{\bar{Q}_a, Q_b\} = D^\kappa_{ab} = (\gamma^0)_{ab} 2\kappa \sinh\frac{P_0}{2\kappa} - (\gamma_i)_{ab} P_i = (\gamma^\mu)_{ab} \tilde{P}_\mu \tag{3b}$$

where $\tilde{P}_i = P_i$, $\tilde{P}_0 = 2\kappa \sinh(P_0/2\kappa)$.

We see that the right-hand side of (3*b*) describes the Dirac root of the κ-deformed mass Casimir C_2 of the κ-Poincaré algebra (see [15])

$$C_2 = \left(2\kappa \sinh\frac{P_0}{2\kappa}\right)^2 - \boldsymbol{P}^2 = 2\kappa^2\left(\cosh\frac{P_0}{\kappa} - 1\right) - \boldsymbol{P}^2 = \tilde{P}_0 - \boldsymbol{P}^2. \tag{4}$$

(ii) In the quantum superalgebra $U_\kappa(\mathcal{P}_{4;1})$ due to the relations (3*b*) one can replace the fourmomenta operators by the bilinears of supercharges. In our framework the κ-Poincaré algebra $U_\kappa(\mathcal{P}_4)$ can be obtained from $U_\kappa(\mathcal{P}_{4;1})$ if we put all four supercharges Q_a equal to zero.
(iii) The deformed Lorentz algebra ceases to be a Hopf subalgebra of $U_\kappa(\mathcal{P}_{4;1})$ because the boost commutators depend on the fourmomenta as well as the bilinears of the supercharges. We have

$$\begin{array}{llll} U_\kappa(\mathcal{P}_{4,1}) \supset & U_\kappa(\mathcal{P}_4) \supset & U_\kappa(O(3,1)) & (\kappa < \infty) \\ \downarrow \kappa = \infty & \downarrow \kappa = \infty & \downarrow \kappa = \infty & \\ U(\mathcal{P}_{4;1}) \supset & U(\mathcal{P}_4) \supset & U(O(3,1)) & (\kappa = \infty). \end{array} \tag{5}$$

In this letter we first describe briefly the real form of $U_q(OSp(1|4))$, which undergoes subsequently the contraction (2). We should mention here that in 1991 the algebraic sector of the κ-super Poincaré algebra was calculated by performing the contraction of $U_q(OSp(1|4))$ for $|q| \to 1$ with non-standard reality condition [12]§. It appears, however, that (compare the discussion of κ-Poincaré algebras—e.g. [13, 15]) the standard real structure $U_q(OSp(1|4))$ which provides, after the contraction limit (2), the flat $O(3)$ sector, is described necessarily by the involution which maps the

† The contraction (2) for $U_q(SU(2))$ was proposed firstly by the Firenze group [21]. It should be also observed the factor 2 occurring in the limit (1.2) in the supersymmetric case providing simple comparison with the reults of [15].

‡ Here we write basic supersymmetry relations in its Majorana form with real four supercharges. In sections 3 and 4 we shall use the two-component Weyl spinor notation.

§ We would like to add here that the relations (5.20) in (5.23) in [12] contain some errors (e.g. in (5.20) the parameter κ should be replaced by 2κ).

standard Cartan–Weyl basis into the elements of the antipode-extended Cartan–Weyl basis. It is this antipode-extended Cartan–Weyl basis of $U_q(OSp(1|4))$ ($S \equiv$ antipode):

$$\text{bosonic:} \quad \begin{matrix} e_{\pm\alpha} & e_{\pm(\alpha+2\beta)} & e_{\pm(2\alpha+2\beta)} & e_{\pm 2\beta} \\ S(e_{\pm(\alpha+2\beta)}) & S(e_{\pm(2\alpha+2\beta)}) & h_\alpha & h_\beta \end{matrix} \tag{6a}$$

$$\text{fermionic:} \quad e_{\pm\beta} \qquad e_{\pm(\alpha+\beta)} \qquad S(e_{\pm(\alpha+\beta)}) \tag{6b}$$

which should be used for the introduction of the physical generators of N = 1 D = 4 Poincaré superalgebra. The standard as well as non-standard Cartan–Weyl basis of $U_q(OSp(1|4))$—the second one chosen from the generators (6*a*, *b*) and adjusted to the real form under consideration—is described in section 2. In section 3 we define the physical generators and after suitable non-linear redefinition of boost generators we write down explicitly the quantum κ-Poincaré superalgebra (the real Hopf algebra) in its final form. One obtains that the κ-supersymmetry algebra (3) is supplemented by the following coproduct formulae

$$\Delta(Q_a) = Q_a \otimes e^{P_0/4\kappa} + e^{-(P_0/4\kappa)} \otimes Q_a \tag{7a}$$

which leads via the relations†

$$\{\Delta(\bar{Q}_a), \Delta(Q_b)\} = (\gamma^\mu)_{ab} \Delta(\tilde{P}_\mu) \tag{7b}$$

to the coproduct

$$\Delta(P_0) = P_0 \otimes 1 + 1 \otimes P_0 \tag{8a}$$

$$\Delta(P_i) = P_i \otimes e^{P_0/2\kappa} + \varepsilon^{-(P_0/2\kappa)} \otimes P_i \tag{8b}$$

describing the four-momentum sector of $U_\kappa(\mathcal{P}_4)$ [15].

In this letter we present the derivation of κ-Poincaré algebra by contraction. The κ-deformation of Poincaré algebra can also be obtained in an algebraic way, following the alternative discussion of κ-Poincaré algebra [16, 17]. The algebraic derivation of κ-Poincaré superalgebra is now under consideration.

Let us introduce first the root system for the superalgebra $B(0,2) \simeq OSp(1|4)$. If the fermionic roots $\delta_1 = \alpha + \beta$, $\delta_2 = \beta$ are endowed with the scalar product

$$(\delta_1, \delta_1) = (\delta_2, \delta_2) = 1 \qquad (\delta_1, \delta_2) = 0 \tag{9a}$$

one obtains the following symmetric Cartan matrix

$$\alpha_{ij} = \begin{pmatrix} (\alpha,\alpha) & (\alpha,\beta) \\ (\beta,\alpha) & (\beta,\beta) \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}. \tag{9b}$$

† From (8*a*) follows that, consistently with the relations (7*b*), one obtains

$$\Delta(\tilde{P}_0) = \tilde{P}_0 \otimes e^{P_0/\kappa} + e^{-(P_0/\kappa)} \otimes \tilde{P}_0$$

where

$$e^{\pm(P_0/\kappa)} = \left(1 + \frac{\tilde{P}_0^2}{4\kappa^2}\right)^{1/2} \pm \frac{\tilde{P}_0}{2\kappa}.$$

The standard q-deformed Cartan–Weyl basis is obtained by adding the non-simple generators defined by the formulae ([6]; $[A, B]_x = AB - xBA$, $\{A, B\}_x = AB + xBA$)

$$
\begin{aligned}
e_{\alpha+\beta} &= [e_\alpha, e_\beta]_{q^{-1}} & e_{-(\alpha+\beta)} &= [e_{-\beta}, e_{-\alpha}]_q \\
e_{\alpha+2\beta} &= \{e_{\alpha+\beta}, e_\beta\} & e_{-(\alpha+2\beta)} &= \{e_{-(\alpha+\beta)}, e_{-\beta}\} \\
e_{2\beta} &= (1+q)e_\beta^2 & e_{-2\beta} &= (1+q^{-1})e_{-\beta}^2 \\
e_{2(\alpha+\beta)} &= (1+q)e_{\alpha+\beta}^2 & e_{-2(\alpha+\beta)} &= (1+q^{-1})e_{-(\alpha+\beta)}^2.
\end{aligned}
\tag{10}
$$

The six generators $e_{\pm\alpha} \equiv e_{\pm 1}$, $e_{\pm\beta} \equiv e_{\pm 2}$, h_i $(i = 1, 2)$ satisfy the Drinfeld–Jimbo relations ([5, 6]; $[x](q - q^{-1})^{-1} \cdot (q^x - q^{-x})$):

$$
\begin{aligned}
&[e_\alpha, e_{-\alpha}] = [h_\alpha] \qquad \{e_\beta, e_{-\beta}\} = [h_\beta] \\
&[e_\alpha, e_{-\beta}] = 0 \\
&[h_i, e_{\pm j}] = \pm \alpha_{ij} e_{\pm j}
\end{aligned}
\tag{11}
$$

as well the q-deformed $OSp(1|4)$ Serre relations. One can derive the following algebraic relations for the 14 generators of Cartan–Weyl basis†:

(a) Fermionic q-deformed bilinear relations (F–F sector)

$$
\begin{aligned}
&\{e_\beta, e_{-(\alpha+\beta)}\} = e_{-\alpha} q^{-h\beta} \\
&\{e_{\alpha+\beta}, e_{-(\alpha+\beta)}\} = [h_\alpha + h_\beta]_q.
\end{aligned}
\tag{12a}
$$

(b) Fermionic q-covariance relations (B–F) sector)

$$
\begin{aligned}
&[e_\alpha, e_{\alpha+\beta}]_q = [e_{\alpha+\beta}, e_{\alpha+2\beta}]_q = [e_{\alpha+2\beta}, e_\beta]_q = 0 \\
&[e_{\alpha+\beta, e-\alpha}] = -e_\beta q^{-h_\alpha} \\
&[e_{\alpha+2\beta}, e_{-(\alpha+\beta)}] = -e_\beta q^{-(h_\alpha+h_\beta)} \\
&[e_\beta, e_{-(\alpha+2\beta)}] = -e_{-(\alpha+\beta)} q^{-h_\beta} \\
&[e_\beta, e_{2\beta}] = [e_{\alpha+\beta}, e_{2(\alpha+\beta)}] = 0 \\
&[e_{2\beta}, e_{-\beta}] = -e_\beta(q^{-h_\beta} + q q^{h_\beta}) \\
&[e_{\alpha+\beta}, e_{2\beta}] = (q - q^{-1}) e_{\alpha+2\beta} e_\beta \\
&[e_{2\beta}, e_{-(\alpha+\beta)}] = (q - q^{-1}) e_\beta e_{-\alpha} q^{-h_\beta} \\
&[e_\beta, e_{2(\alpha+\beta)}] = (q^{-1} - q) e_{\alpha+\beta} e_{\alpha+2\beta} \\
&[e_{2(\alpha+\beta)}, e_{-\beta}] = (q^{-2} - 1) e_\alpha e_{\alpha+\beta} q^{h_\beta} \\
&[e_{2(\alpha+\beta)}, e_{-(\alpha+\beta)}] = -(q e_{\alpha+\beta} q^{h_\alpha+h_\beta} + e_{\alpha+\beta} q^{-(h_\alpha+h_\beta)}).
\end{aligned}
\tag{12b}
$$

(c) Bosonic q-deformed bilinear relations (B–B sector)

$$
\begin{aligned}
&[e_\alpha, e_{\alpha+2\beta}] = e_{2(\alpha+\beta)} \\
&[e_{\alpha+2\beta}, e_{-\alpha}] = -e_{2\beta} q^{-h_\alpha} \\
&[e_{\alpha+2\beta}, e_{-(\alpha+2\beta)}] = -[h_\alpha + 2h_\beta]_q
\end{aligned}
$$

† We present only half of the algebraic relations. The other half can be obtained by the following anti-automorphism

$$
e_{\pm i} \rightarrow e_{\mp i} \qquad h_i \rightarrow h_i \qquad q \rightarrow q^{-1}
$$

called in [6] the Cartan–Planck involution.

$$[e_{2\beta}, e_{-2\beta}] = (q-q^{-1})^2 e_\beta e_{-\beta}[h_\beta]_q + (1-q^{-2h_\beta}) - (1+q)[2h_\beta]_q$$

$$[e_{2(\alpha+\beta)}, e_{-2(\alpha+\beta)}] = 1 - q^{-2(h_\alpha+h_\beta)} + e_{\alpha+\beta}e_{-(\alpha+\beta)}(q-q^{-1})^2 \cdot [h_\alpha + h_\beta]_q - (1+q)[2h_\alpha + 2h_\beta]_q$$

$$[e_\alpha, e_{2\beta}]_{q^{-2}} = (1+q^{-1})e_{\alpha+2\beta} + (q-q^{-1})e_{\alpha+\beta}e_\beta \tag{12c}$$

$$[e_{2\beta}, e_{-\alpha}] = [e_{\alpha+2\beta}, e_{2\beta}]_{q_2} = 0$$

$$[e_{2\beta}, e_{-(\alpha+2\beta)}] = -(q-q^{-1})e_\beta e_{-(\alpha+\beta)}q^{-h_\beta} - (1+q^{-1})e_{-\alpha}q^{-2h_\beta}$$

$$[e_\alpha, e_{2(\alpha+\beta)}]_{q^2} = [e_{2(\alpha+\beta)}, e_{\alpha+2\beta}]_{q^2} = 0$$

$$[e_{2(\alpha+\beta), e-\alpha}] = -(q-q^{-1})e_{\alpha+\beta}e_b q^{-h_\alpha} - (1+q^{-1})e_{\alpha+2\beta}q^{-h_\alpha}$$

$$[e_{2(\alpha+\beta)}, e_{-(\alpha+2\beta)}] = (q^2-1)e_{\alpha+\beta}e_{-\beta}q^{h_\alpha+h_\beta} - (1+q)e_\alpha q^{h_\alpha+2h_\beta}$$

$$[e_{2(\alpha+\beta)}, e_{2\beta}] = (1+q)(q-q^{-1})e^2_{\alpha+2\beta} + (q-q^{-1})^2 e_{\alpha+\beta}e_{\alpha+2\beta}e_\beta$$

$$[e_{2(\alpha+\beta)}, e_{-2\beta}] = (1+q^{-1})(q^{-2}-1)q^{-1}e^2_\alpha q^{2h_\beta} + (q^{-2}-1)^2 e_\alpha e_{\alpha+\beta}e_{-\beta}q^{h_\beta}.$$

The algebraic relations (12*a*–*c*) should be supplemented by the coproducts derived from the Drinfeld–Jimbo classical formulae for simple root generators [1, 5, 6]:

$$\Delta(e_{\pm i}) = e_{\pm i} \otimes q^{h_i/2} + q^{-(h_i/2)} \otimes e_{\pm i} \tag{13}$$

by using the relation $\Delta(ab) = \Delta(a) \cdot \Delta(b)$ where $a, b \in U_q(OSp(1|4))$. We would like to stress here that in this derivation the following formula [6]:

$$(a \otimes b) \cdot (c \otimes d) = (-1)^{\mathrm{grad}\, b \cdot \mathrm{grad}\, c}(ac) \otimes (bd) \tag{14}$$

should be used. Similarly, by using the relation $S(ab) = (-1)^{\mathrm{grad}\, a \cdot \mathrm{grad}\, b} S(b)S(a)$ and the formulae [15, 6]

$$S(e_{\pm\alpha}) = -q^{\pm 1}e_{\pm\alpha} \qquad S(e_{\pm\beta}) = -q^{\pm(1/2)}e_{\pm\beta} \qquad S(h_i) = -h_i \tag{15}$$

one obtains the antipodes for all 14 generators of the standard q-deformed Cartan–Weyl basis. We shall use further the following involution

$$e_i^+ = e_{-i} \qquad h_i^+ = h_i. \tag{16}$$

With such an involution the following two choices of the deformation parameter q can be considered:

(i) $|q| = 1$. In such a case the involution (16*a*) does not take us out of the Cartan–Weyl basis of $U_q(OSp(1|4))$, but the coproduct formulae are only invariant if the involution acts on tensor products in a non-standard way ($(a \otimes b)^+ = b^+ \otimes a^+$). Therefore for q on the unit circle we obtain $U_q(OSp(1|4))$ as a non-standard real Hopf superalgebra, and such a choice was investigated in [12].

(ii) q real. This choice is proposed in the present paper and it leads to a standard real Hopf superalgebra $U_q(OSp(1|4))$. The technical complication is due to the appearance of the generators $\tilde{e}_{-A}$ (see $6a, b$) generated by the involution ($16a$). More explicitly, by using the relations $(ab)^+ = b^+ a^+$ one obtains

$$\begin{aligned}
e^+_{\alpha+\beta} &= \tilde{e}_{-(\alpha+\beta)} = q^{-2} e_{-(\alpha+\beta)} + (1-q^{-2}) e_{-\beta} e_{-\alpha} \\
e^+_{\alpha+2\beta} &= \tilde{e}_{-(\alpha+2\beta)} = q^{-2} e_{-(\alpha+2\beta)} + (1-q^{-2}) e_{-2\beta} e_{-\alpha} - (q^{-1}-q^{-3}) e_{-\beta} e_{-(\alpha+\beta)} \\
e^+_{2\beta} &= \tilde{e}_{-2\beta} = q e_{-2\beta} \qquad (16b) \\
e^+_{2(\alpha+\beta)} &= \tilde{e}_{-2(\alpha+\beta)} = q^{-3} e_{-2(\alpha+\beta)} + (1-q^{-2})^2 e_{-2\beta} e^2_{-\alpha} \\
&\quad + (1+q) q^{-2} (1 - q^{-2} e_{-(\alpha+2\beta)} e_{-\alpha}) - q^{-1} (1-q^{-2})^2 e_{-\beta} e_{-(\alpha+\beta)} e_{-\alpha}.
\end{aligned}$$

The set of generators

$$e_{\pm 1} \qquad h_i \qquad e_A \qquad \tilde{e}_{-A} \qquad (A = \alpha+\beta, \alpha+2\beta, 2\beta, 2(\alpha+\beta)) \qquad (17)$$

is invariant under the involution ($16a, b$) and defines a non-standard q-deformed Cartan–Weyl basis of $U_q(OSp(1|4))$.

The set (17) replaces, for q real, the standard Cartan–Weyl basis with non-simple generators (10). The relations ($12a$–c) should be modified by removing all the generators e_{-A} and inserting the generators $\tilde{e}_{-A}$. The only new relations which should be calculated are $[e_a, \tilde{e}_{-b}]$, where $a, b = (\alpha+\beta, \alpha+2\beta, 2(\alpha+\beta))$†—the other (in particular $[\tilde{e}_{-a}, \tilde{e}_{-b}]$) can be obtained from the ones given by ($12a$–c) by the action of antipode or the involution ($16a$). For example we obtain the following 'diagonal' commutator‡

$$\{e_{\alpha+\beta}, \tilde{e}_{-(\alpha+\beta)}\} = q^{-2} [h_\alpha + h_\beta]_q + (1-q^{-2}) (e_\alpha e_{-\alpha} q^{h_\beta} - e_\beta e_{-\beta} q^{-h_\alpha}) + q^{-1} (q^{h_\beta - h_\alpha} - q^{-(h_\alpha + h_\beta)}) \qquad (18)$$

and the following 'non-diagonal' one

$$\begin{aligned}
[e_{\alpha+\beta}, \tilde{e}_{-(\alpha+2\beta)}] &= -e_\beta q^{-(h_\alpha+h_\beta)} - (q-q^{-1}) e_\beta q^{h_\beta - h_\alpha} - (q-q^{-1}) e_{\alpha+\beta} e_{-\alpha} q^{h_\beta} \\
&\quad + (q^{-2}-1) e_{2\beta} e_{-\beta} q^{-h_\alpha}
\end{aligned}$$

which should be compared with the second formula in ($12a$) and the third one in ($12b$) Similarly using the formulae (16) one can calculate the coproducts $\Delta(\tilde{e}_{-A})$ as well as the antipodes $S(\tilde{e}_{-A})$.

† From ($16b$) it follows that $\tilde{e}_{-2\beta}$ is proportional to $e_{-2\beta}$.
‡ Non-standard Cartan–Weyl basis can be defined as providing diagonal commutators which are not entirely functions of Cartan generators.

We introduce below 14 linear combinations of generators (17) which are self-conjugate under the involution (16*a*, *b*):

(a) Lorentz generators

$$M_{12}=\tfrac{1}{2}h_\alpha \qquad M_{23}=\tfrac{1}{2}(e_\alpha+e_{-\alpha}) \qquad M_{31}=\frac{-\mathrm{i}}{2}(e_\alpha-e_{-\alpha})$$

$$\begin{aligned} M_{10}&=\frac{\mathrm{i}}{4}(e_{2(\alpha+\beta)}-\tilde{e}_{-2(\alpha+\beta)}-e_{2\beta}+\tilde{e}_{-2\beta})\\ M_{20}&=\tfrac{1}{4}(e_{2\beta}+\tilde{e}_{-2\beta}+e_{2(\alpha+\beta)}+\tilde{e}_{-2(\alpha+\beta)})\\ M_{30}&=\frac{\mathrm{i}}{2}(\tilde{e}_{-(\alpha+2\beta)}-e_{\alpha+2\beta}).\end{aligned} \tag{19a}$$

We shall use also the notation

$$\begin{aligned} &M_3=M_{12} \qquad M_\pm=M_{23}\pm\mathrm{i}M_{31}\\ &L_3=M_{30} \qquad L_\pm=M_{10}\pm\mathrm{i}M_{20}.\end{aligned} \tag{19b}$$

(b) Curved translations

$$\begin{aligned} M_{51}&=\tfrac{1}{4}(e_{2\beta}+\tilde{e}_{-2\beta}-e_{2(\alpha+\beta)}-\tilde{e}_{-2(\alpha+\beta)})\\ M_{52}&=\frac{\mathrm{i}}{4}e_{2\beta}-\tilde{e}_{-2\beta}+e_{2(\alpha+\beta)}-\tilde{e}_{-2(\alpha+\beta)})\\ M_{53}&=\tfrac{1}{2}(e_{\alpha+2\beta}+\tilde{e}_{-(\alpha+2\beta)})\\ M_{50}&=\tfrac{1}{2}(h_\alpha+2h_\beta).\end{aligned} \tag{20}$$

(c) Supercharges (in two-component Weyl notation):

$$\begin{aligned} &\Psi_1=e_\beta-\tilde{e}_{-(\alpha+\beta)} \qquad \bar{\Psi}_1=e_{-\beta}-e_{\alpha+\beta}\\ &\Psi_2=-(e_{\alpha+\beta}+e_{-\beta}) \qquad \bar{\Psi}_2=-(e_\beta+\tilde{e}_{-(\alpha+\beta)}).\end{aligned} \tag{21}$$

If $q=1$ ten real generators M_{KL} ($K=1,2,3,0,5$; $M_{KL}=M_{KL}^+$) describe the $O(3,2)$ algebra with metric $g_{KL}=\mathrm{diag}(-1,\ -1,\ -1,1,1,)$, and four generators (21) describe four $OSp(1|4)$ supercharges, satisfying the reality conditions $\Psi_A^+=\bar{\Psi}_{\dot{A}}$. In order to contract $OSp(1|4)$ to $\mathrm{N}=1$, $\mathrm{D}=4$ Poincaré algebra one redefines the generators (20) and (21) as follows ($\mu=1,2,3,0$)

$$\begin{aligned} &M_{5\mu}=RP_\mu\\ &\Psi_A=R^{1/2}Q_A \qquad \bar{\Psi}_{\dot{A}}=R^{1/2}\bar{Q}_{\dot{A}}\end{aligned} \tag{22}$$

and performs the limit $R\to\infty$.

The formulae (22) as well as the rescaling (2) of the deformation parameter q lead to the contraction of the q-deformed real $U_q(OSp(1|4))$ quantum algebra. In order to obtain in the contraction limit our κ-deformed Poincaré superalgebra we should:
(a) describe all the commutators

$$[M_{KL}, M_{ST}] \qquad [M_{KL}, Q_A] \qquad [M_{KL}, \bar{Q}_{\dot{A}}]$$

and the anticommutators of any two supercharges Q_A, $\bar{Q}_{\dot{A}}$ in terms of the generators (17), satisfying the relations (12*a*–*c*)
(b) perform the contraction (2)
(c) follow our experience with κ-Poincaré algebra [14, 15] to find the non-linear transformation of contracted generators which simplifies the result.

In our calculations we shall define the following new boost generators ($r=1,2$)

$$\begin{aligned} \tilde{L}_\pm &= L_\pm \mp \frac{\mathrm{i}}{4\kappa}(P_\pm + \{M_\pm, P_3\}) \\ \tilde{L}_3 &= L_3 + \frac{\mathrm{i}}{4\kappa}(M_+P_- - P_+M_-) + \frac{\mathrm{i}}{8\kappa}(Q_2Q_1 - \bar{Q}_1\bar{Q}_2). \end{aligned} \tag{23}$$

(d) Using the formulae (19–21) one should calculate the limit (2) of coproducts $\Delta(M_{KL})\Delta(Q_A)$, $\Delta(\bar{Q}_{\dot{A}})$ and of the antipodes.

The results can be described by the following set of relations:
(a) Non-relativistic $O(3) \supset T_4$ sector (M_i, P_i, P_0).
(i) Algebra

$$\begin{aligned} &[M_i, M_j] = \mathrm{i}\varepsilon_{ijk} M_k \\ &[M_i, P_j] = \mathrm{i}\varepsilon_{ijk} P_k \qquad [M_i, P_0] = 0 \\ &[P_\mu, P_\nu] = 0. \end{aligned} \tag{24}$$

(ii) Coalgebra

$$\Delta(M_i) = M_i \otimes 1 + 1 \otimes M_i. \tag{25}$$

The coproducts for $P_\mu = (P_i, P_0)$ are given by the formulae (8*a*, *b*).
(iii) Antipodes:

$$S(M_i) = -M_i \qquad S(P_\mu) = -P_\mu. \tag{26}$$

(b) Boost sector (L_i)
(i) Algebra

$$
\begin{aligned}
&[\tilde{L}_i, M_j] = \mathrm{i}\varepsilon_{ijk}\tilde{L}_k \\
&[\tilde{L}_i, \tilde{L}_j] = -\mathrm{i}\varepsilon_{ijk}\left(M_k \cosh\frac{P_0}{\kappa} - \frac{1}{8\kappa}T_k \sinh\frac{P_0}{2\kappa} + \frac{1}{16\kappa^2}P_k(T_0 - 4MP)\right) \\
&[\tilde{L}_i, P_j] = \mathrm{i}\kappa\delta_{ij}\sinh\frac{P_0}{\kappa} \\
&[\tilde{L}_i, P_0] = \mathrm{i}P_i
\end{aligned}
\tag{27}
$$

where

$$
T_\mu = Q^A(\sigma_\mu)_{A\dot{B}}\bar{Q}^{\dot{B}} \qquad \sigma_\mu = (\boldsymbol{\sigma}, \mathbb{1}_2) \tag{28a}
$$

and

$$
\begin{aligned}
&[T_i, T_j] = -4\mathrm{i}\varepsilon_{ijk}\left(P_k T_0 - 2\kappa\sinh\frac{P_0}{2\kappa}T_k\right) \\
&[T_0, T_i] = -4\mathrm{i}\varepsilon_{ijk}P_j T_k.
\end{aligned}
\tag{28b}
$$

(ii) Coalgebra

$$
\begin{aligned}
\Delta(\tilde{L}_i) = {}& \tilde{L}_i \otimes e^{P_0/2\kappa} + e^{-(P_0/2\kappa)}\tilde{L}_i + \frac{1}{2\kappa}\varepsilon_{ijk}(P_j \otimes M_k e^{P_0/2\kappa} + M_j e^{-(P_0/2\kappa)} \otimes P_k) \\
& + \frac{\mathrm{i}}{8\kappa}(\sigma_i)_{\dot{A}B}(\bar{Q}_{\dot{A}}e^{-(P_0/4\kappa)} \otimes Q_B e^{P_0/4\kappa} + Q_B \mathrm{e}^{-(P_0/4\kappa)} \otimes \bar{Q}_{\dot{A}} e^{P_0/4\kappa}).
\end{aligned}
\tag{29}
$$

(iii) Antipodes

$$
S(\tilde{L}_i) = -\tilde{L}_i + \frac{\mathrm{i}}{\kappa}P_i = -\tilde{L}_i + \frac{3\mathrm{i}}{2\kappa}P_i - \frac{\mathrm{i}}{8\kappa}(Q\sigma_i\bar{Q} + \bar{Q}\sigma_i Q). \tag{30}
$$

(c) Supercharges sector
(i) Algebra

$$
\begin{aligned}
&\{Q_A, \bar{Q}_{\dot{B}}\} = 4\kappa\delta_{A\dot{B}}\sinh\frac{P_0}{2\kappa} - 2P_i(\sigma_i)_{A\dot{B}} \\
&\{Q_A, Q_B\} = \{\bar{Q}_{\dot{A}}, \bar{Q}_{\dot{B}}\} = 0 \\
&[M_i, Q_A] = -\tfrac{1}{2}(\sigma_i)_A{}^B Q_B \\
&[\tilde{L}_i, Q_A] = -\frac{\mathrm{i}}{2}\cosh\frac{P_0}{2\kappa}(\sigma_i)_A{}^B Q_B \\
&[P_\mu, Q_A] = [P_\mu, \bar{Q}_{\dot{A}}] = 0.
\end{aligned}
\tag{31}
$$

(ii) Coproducts

$$
\begin{aligned}
&\Delta(Q_A) = Q_A \otimes e^{P_0/4\kappa} + e^{-(P_0/4\kappa)} \otimes Q_A \\
&\Delta(\bar{Q}_{\dot{A}}) = \bar{Q}_{\dot{A}} \otimes e^{P_0/4\kappa} + e^{-(P_0/4\kappa)} \otimes \bar{Q}_{\dot{A}}.
\end{aligned}
\tag{32}
$$

In four-component notation

$$Q_a = \begin{pmatrix} Q_A + \bar{Q}_{\dot{A}} \\ \mathrm{i}(Q_A - \bar{Q}_{\dot{A}}) \end{pmatrix}$$

we obtain the formulae (1.7*a*).

(iii) Antipodes

$$S(Q_A) = -Q_A \qquad S(\bar{Q}_{\dot{A}}) = -\bar{Q}_{\dot{A}}. \tag{33}$$

We would like to add here that—due to the supersymmetry relation (31)—one can replace the four-momenta by the supercharges

$$P_i = \tfrac{1}{4}(Q\sigma_i\bar{Q} + \bar{Q}\sigma_i Q) \tag{34a}$$

$$\sinh\frac{P_0}{2\kappa} = \frac{1}{8\kappa}\{Q_A, \bar{Q}_{\dot{A}}\}. \tag{34b}$$

In the formulae (24)–(33) this freedom has been fixed by the assumption that by putting in the *bosonic* quantum Poincaré algebra sector (i.e. in the relations (24)–30)) $Q_A = \bar{Q}_{\dot{A}} = 0$ we obtain the known κ-Poincaré algebra $U_q(\mathcal{P}_4)$ introduced in [15]. It should be stressed that putting $Q_A = \bar{Q}_{\dot{A}} = 0$ and leaving $P_\mu \neq 0$ is inconsistent in the *fermionic* sector (see (31)). We would like to point out that for the algebraic relations (24), (27), (31) the Jacobi identities have been verified, and it has been shown that the coproduct Δ: $U_\kappa(\mathcal{P}_{4;1}) \to U_\kappa(\mathcal{P}_{4;1}) \otimes U_\kappa(\mathcal{P}_{4;1})$, described by the formulae (8*a*, *b*), (25), (29) and (32) is the algebra homomorphism.

Following the formulae for the Casimirs of N = 1, D = 4 Poincaré algebra (see [22], p. 72) we obtain the following two invariants of $U_\kappa(\mathcal{P}_{4;1})$:

(a) Quadratic mass Casimir

The mass Casimir for $U_\kappa(\mathcal{P}_{4;1})$ is the same as for $U_q(\mathcal{P}_4)$ and is given by (4).

(b) Fourlinear spin Casimir

The generalized Pauli–Lubanski vector for N = 1, D = 4 supersymmetry

$$\begin{aligned} W_i &= M_i P_0 + \varepsilon_{ijk} P_j L_k + T_i \\ W_0 &= PM + T_0 \end{aligned} \tag{35}$$

defines the superspin s by means of the formulae (for mass Casimir P^2 with eigenvalue m^2)†

$$C_2 = W^2 - \frac{1}{4P^2}(P\cdot W)^2 = -m^2 s(s+1). \tag{36}$$

The κ-extension of the formulae (35, 36) is now under consideration.

In this paper we have generalized the contraction scheme, providing κ-Poincaré algebra to the N = 1 Poincaré superalgebra. It is also natural to generalize several developments, which followed the calculation of κ-deformed Poincaré algebra. In particular

(i) One can extend the fourmomentum space relations of κ-Poincaré algebra [15, 16, 23, 24] to the superspace relations

$$\underset{\kappa\text{-Poincaré}}{\varphi_A(p_\mu)} \;\to\; \underset{N=1\ \kappa\text{-super Poincaré}}{\varphi_A(p_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})} \tag{37}$$

† For $P^2 = 0$ also $P\cdot W = 0$ and the superhelicity is defined if we add to 14 superPoincaré generators the additional one, describing by its eigenvalues the chirality. Such a generator occurs naturally in the superconformal framework.

and discuss the κ-deformation of supersymmetric field multiplets.
(ii) Recently the classical r-matrix ($R=1+\frac{1}{\kappa}r+O(\frac{1}{\kappa^2})$)

$$r(U_\kappa(\mathcal{P}_4))=L_i\wedge P_i \tag{38}$$

has been obtained by Zakrzewski [25] from the κ-deformed Poincaré algebra, given in [15]. Similarly, calculating the classical limit of $U_\kappa(\mathcal{P}_{4;1})$ as a Lie bi-superalgebra with cocommutator describing the antisymmetric part of the coproducts (29), (32) and (8a, b) (linear in $1/\kappa$) one obtains

$$r(U_\kappa(\mathcal{P}_{4;1}))=L_i\wedge P_i-\frac{\mathrm{i}}{4}Q_A\wedge\bar{Q}_{\dot{A}}. \tag{4.3}$$

Following [25] one can obtain the quantum κ-super Poincaré group by a quantization of the graded Poisson structure generated by (39).

The κ-superPoincaré group described by the classical r-matrix (39) describes the quantum deformation of N = 1 D = 4 superPoincaré group in the lowest order in $1/\kappa$.

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